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ABSTRACT

This study uses an algorithm-based visual display technique (FACES) to provide enhanced detection of multivariate outliers within large-scale data sets. The FACES computer graphing algorithm (H. Chernoff, 1973) constructs a cartoon-like face, using up to 18 variables for each case. A major advantage of FACES is the ability to store and show the values of the variables. The research used data from a national, longitudinal study of school children, their parents, and their teachers, and the National Education Longitudinal Study of 1988 (NELS-88). Data for the Stanford Achievement tests from a Florida school district were also used. Four random stratified samples of 250 cases each were drawn from male and female databases for the NELS-88 and Stanford scores. Pearson product-moment correlation coefficients were calculated between the variables in each sample, and variables were assigned to the features of the FACE. The application of the FACES graphing algorithm translates the data into a meaningful visual correlate, providing one summative visual image per student. The ease of outlier detection on seeing a page of Chernoff FACES is readily apparent to the trained researcher, the school administrator, and parents. Four appendixes contain the Mahalanobis distance values for faces for the four samples. Also attached is a set of FACES for each sample that illustrates outlier cases. (SLD)

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Enhanced Detection of Multivariate Outliers Using Algorithm-Based Visual Display Techniques

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Enhanced Detection of Multivariate Outliers Using Algorithm-Based Visual Display Techniques

Wendy B. Dickinson

Introduction: Importance of Outlier Detection

As Stevens (1990) relates, statistical procedures can be quite sensitive to outliers. This is especially true for the multivariate procedures that are being considered here. As researchers, it is crucially important to be able to identify those outliers. It is also important to be able to make informed, appropriate decisions about both their origin and disposition. Because even one outlier can have a deleterious effect on the outcome of data analysis, it is necessary to look at the data and determine if indeed an aberrant observation is an outlier or just a case of bad typing skills during data entry. If the observation value has been correctly entered, and is still markedly deviant from the other observations, then it is indeed an actual outlier. If it is determined that the outlier observed is deviant because of an instrumentation error, or that the process followed to generate data for this particular case was different, then it is considered legitimate to drop the outlier, but if it cannot be determined which of these two scenarios is operating (bad data entry versus bad measurement/instrumentation) then the researcher can run the analysis both with and without the outlier value and report the results of both analyses (Stevens, 1990).

> Outliers, whether they are data recording and data entry mistakes or truly unusual observations, can wreak havoc on both parameter estimation and statistical inference. Whether outliers ought to be omitted or included in the analysis is a matter of judgement that depends on the substantive topic, the cause of the outlier, and other issues. But failure to detect and identify outliers whether or not they are included in the analysis is scientifically irresponsible.
> (Judd & McClelland, 1989, 237.)

"Outliers should not...be regarded as "bad"...is has been argued that outliers can provide

some of the most interesting cases of further study" (Stevens, 1990.p. 18).

Herman Chernoff (1973) developed a computer graphing algorithm that constructed a cartoon-like FACE using up to 18 variable values for each case. Jacob (1978) stated that Chernoff FACES combined "a number of desirable properties for representing multivariate data graphically, including the integration of display dimensions and the general familiarity of faces.

A major advantage of the FACES program is the capability to store and show the values of up to 18 different variables. This extra capacity for storage and graphical display was a great aid to correctly interpreting the outlier findings. The development of communicative, reproducible multivariate graphing algorithms are essential to efficient, appropriate data analysis. This research contributed both aesthetic suggestions and new programming techniques to aid in the detection and display of multivariate outliers.

Purpose

The purpose of this study was to utilize an algorithm-based visual display technique (FACES) to provide enhanced detection of multivariate outliers within large-scale data sets.

Data Sources

This research utilized data from a national, longitudinal research study of school children, their parents, and their teachers; and data from a local Florida school district. The source of the national research data was the NELS-88 data base. NELS-88 is a longitudinal, national research project that sampled from the population of 8th grade students in the United States. Major objectives and features of the study include the integration of student, parent, teacher, and school studies; an initial concentration on the eight-grade cohort with follow-up studies planned at successive two-year intervals; the inclusion of supplementary components designed to support analyses of both geographically and demographically distinct subgroups; and the design linkages

longitudinal studies and additional current studies (National Center for Education Statistics, 1990). Underlying all these individual features is a central theme: "education in America must be understood as a lifelong process enmeshed in a complex social context" (NCES, 1990, p. 5). Selected sections of the NELS-88 database were extracted for analysis. The source of the local data was student scores on the Stanford Achievement tests. The student scores were randomly selected from a large, urban public school district database. The Stanford data were extracted from the school district mainframe using SPSS programing. Using a random number generator, 250 male and 250 female cases were randomly selected from the 8th grade Stanford test scores. No additional demographic information was provided by the school district. From each database (NELS-88 and Stanford) two samples were drawn with 250 cases per sample as shown below.

Table 1

NELS-88 and Stanford Database Samples

Gender	NELS-88	Stanford
Male Students	NELS- Males (250 cases)	Stanford-Males (250 cases)
Female Students	NELS-Females (250 cases)	Stanford-Females (250 cases)

Method

Four stratified random samples were drawn from the NELS-88 and Stanford test score databases. Using SAS 6.12, Pearson-product moment correlation coefficients were calculated between the variables in each sample. Variables were assigned to the features of the FACE utilizing these r values. The features of the "Faces" corresponded to individual observations on each variable. The 18 descriptive variables are referred to as X_1 - X_{18}. Each variable value affects the final image and expression of the "Face." The size and placement of the eye is determined by variables X_1 - X_5. The size of the "Face" is mainly shown by the values of X_{13}, the

radius length (OP) to the corner of the face. X_{13} describes the angle of OP (radius) to a horizontal line. X_{13} decides the vertical size of the face: taller (thinner) or shorter (rounder). X_{11} and X_{12} determine the "eccentricity" of the upper and lower hair line. Eccentricity refers to departures from normal facial features and shapes. An example of a "Faces" eccentricity would be a triangle (pointy) chin or a pointed top (cone head). X_{16} describes the length of the nose, while X_{17} and X_{18} describe, respectively, the vertical position, curvature, and width of the mouth. X_1, X_2, X_3, X_4 and X_5 described the vertical positions of the eyes, the eye separation, eccentricity of the eyes, and the size of the eyes. X_3 positions the pupils, and X_9 calibrates the vertical position of the eyebrows. The values of variables X_{17} and X_{18} determined the slant and the size of the mouth.

Using SAS programming code written to calculate Mahalanobis Distance values, the researcher examined the range of values observed in each sample. The researcher used a multivariate, multidimensional graphing algorithm (Chernoff's FACES) to make visual representations of these data. Multivariate outlier detection activities were conducted and effectiveness of the information display was considered by the researcher. Results are presented in the form of comparative displays and Mahalanobis Distance values.

Mahalanobis Distance values were calculated for each case in the four samples. For the Stanford Male and Stanford Female samples, the critical value of Mahalanobis Distance which indicated a multivariate outlier was 18.08. Therefore, any case which had a Mahalanobis Distance greater than or equal to 18.08 was indeed an outlier. Nineteen outliers were identified from the Stanford Female sample of 250 cases. The same mathematical procedure was performed for the NELS-Male and NELS-Female samples. The critical value calculated for Mahalanobis Distance to indicate a multivariate outlier was 32.87. Twenty-three cases were

identified as multivariate outliers from the NELS- Male sample of 250 cases while 24 cases were identified as multivariate outliers from the NELS-Female sample.

Numerical identification of outliers using the Mahalanobis Distance Values was completed. SAS 6.12 programming was used to draw the FACES. Comparison between cases was performed as each FACE is drawn separately. These "FACES" were assembled into a composite for identification of outliers and unusual cases. The numerically identified outliers were then graphed using the FACES algorithm. The algorithm-based FACES were examined to aid as an outlier detection 'tool' for the educational researcher. Each sample yielded 250 of these graphed FACES. One thousand FACES, in total, were graphed for these four samples. As shown, the FACE which corresponded to the minimum Mahalanobis distance, the maximum Mahalonobis distance, and the median Mahalonobis distance were displayed for the NELS-Males, NELS-Females, and the Stanford-males and Stanford Females.

Investigating Multivariate Normality

Rencher (1995) writes, "Checking for multivariate normality is conceptually not as straight forward as assessing univariate normality, and consequently the state of the art is not as well developed." (p. 110). For example, when assessing univarate (one-dimensional) normality, we divide up the sample data into sequential, linear intervals and determine how many observations fall in each interval. With multivariate data, however, we have more than one dimension in our data; and the data, having more dimensions, have more room to spread out (disperse). Consequently, trying to assign multivariate sample data observations is more difficult because there are more possible places the observations could be. Luckily for researchers,

Mardia (1970) developed two procedures for assessing multivariate normality: 1) the skewness index, and (2) the kurtosis index for multivariate populations.

Results and Discussion

Shown in Table 2, Multivariate Skewness and Kurtosis, are the values calculated for each data sample used in this research.

Table 2

Multivariate Skewness and Kurtosis: Mardia's Indices

Sample	Mardia's Skewness Index		Mardia's Kurtosis Index	
	$B_{1,p}$	p	$B_{2,p}$	p
NELS - Males	63.22	.001	487.92	.001
NELS - Females	63.36	.001	507.72	.001
Stanford - Males	15.45	.001	125.58	.001
Stanford - Females	11.65	.001	114.86	.001

As shown in Table 2, the NELS: Male sample and NELS: Female sample have large skewness indices (63.22, 63.36 respectively) and extremely large kurtosis indices (487.92, 507.72 respectively). The reason the indices are so very large is that there are a number of variables entering the calculations (high dimensional) and the population data are significantly non-normal which we would expect to see given the Likert-type responses scale. Because the kurtosis indices are extremely large positive values, (NELS-Males = 487.92, NELS-Female = 507.72), these distributions have a higher peak than a normal distribution, with values clumped together close to the mean, and with a heavier tail.

As for the Stanford samples, the Stanford Male sample is both more skewed (15.45) and

has greater kurtosis (125.58) than the Stanford Female sample (with values of 11.65 and 114.86, respectively, for skewness and kurtosis). Looking at the Stanford samples, again there is positive skewness (Stanford-Males = 15.45, Stanford-Females = 11.65) and marked positive kurtosis (Stanford-Males = 125.58; Stanford-Females = 114.86).

The marked positive kurtosis indicates the heavier tails contain more extreme observations than would be expected. While outliers would be expected to be seen at the .05 level in a population, these samples contained approximately 10% outliers. To assess multivariate normality, the researcher used the calculated values for multivariate skewness ($b_{1,p}$) and multivariate kurtosis ($b_{2,p}$) to enter into Rencher's (1995) approximate test formula. The multivariate skewness and kurtosis indices were statistically significant for all samples.

Mahalanobis' Distance: Multivariate "Yardstick"

Mahalanobis' Distance can best be described as a multivariate "yardstick" used to assess multivariate differentness. Mahalanobis' Distance is a Euclidean-based measure of how far (distant) an observation is from the centroid (multivariate group mean). Because the value of Mahalanobis' Distance of any given observation represents its proximity to the group mean value (centroid), the bigger the Mahalanobis distance the more unusual the observation is. If we think of the centroid value as the "typical" case, then a case with an extremely large Mahalanobis distance would be classified as an "outlier."

The range of Mahalanobis Distance possible values is bounded by 0 on the lower end and ∞ on the upper end. As Rencher (1995) states, "there are two principal approaches for dealing with outliers.... identification and accommodation" (p. 115). Using SAS programming code written to calculate Mahalanobis Distance values, each case in each sample was evaluated. The

next step was to examine the range of values observed in each sample. Table 3 reports the calculated Mahalanobis Distance ranges for each of the four samples. Each sample shows quite a bit of variability, as evidenced by the dispersion of minimum and maximum Mahalanobis Distance values.

The researcher calculated the Mahalanobis Distance for each case in each of the four samples. Appendix 1 exhibits the Mahalanobis Distance Values of the NELS Males sample. Appendix 2 exhibits the Mahalanobis Distance Value of the NELS Female sample. Appendices 3 and 4, respectively, show the Mahalanobis Distance values for the Stanford Males sample and Stanford Females sample.

Outlier Identification in Samples

The Mahalanobis Distance value was calculated for each case in all samples. Using Rencher's (1995) suggested strategy for multivariate outlier identification, the researcher calculated the critical value for Mahalanobis Distance based on a procedure for checking multivariate normalitiy. Using theYang and Lee (1987) E-test formula for checking multivariate normality, a critical value for Mahalanobis Distance was calculated.

The same mathematical procedure was performed for the NELS-Male and NELS-Female samples. The critical value calculated for Mahalanobis Distance to indicate a multivariate outlier is 32.87. In Table 4, cases identified as multivariate outliers from the NELS Male sample of 250 cases are shown. In total, 23 cases were identified as multivariate outliers from the NELS-Male sample. Table 5 shows the NELS - Female sample results. In total, 24 cases were identified as outliers from the 250 cases of the NELS - Female sample.

Table 3

Mahalanobis Distance: Sample Ranges

Sample	Minimum Mahalanobis Distance Range Value	Maximum Mahalanobis Distance RangeValue
NELS - Males	4.39	57.64
NELS - Females	3.72	55.13
Stanford - Males	0.73	59.34
Stanford - Females	0.87	46.90

For visual comparison purposes, the complete identified outlier sets from each of the four samples (NELS-Males, NELS-Females, Stanford Males, and Stanford Females) are shown in Appendices 5, 6, 7, and 8 respectively.

Table 4

Cases Identified as Outliers: NELS Male Sample

Case Number	Mahalanobis Distance Value
69	57.64
171	49.41
44	47.45
52	45.75
207	44.28
65	43.02
70	42.51
98	42.15
153	42.13
11	40.31
244	39.71
99	39.37
175	39.34
102	38.49
202	37.65
134	37.22
100	36.64
185	36.28
165	35.84
86	35.32
111	33.71
169	33.38
94	32.95

Table 5

Cases Identified as Outliers: NELS - Female Sample

Case Number	Mahalanobis Distance Value
182	58.86
4	55.13
61	47.78
22	46.67
73	44.87
62	44.46
144	43.03
113	42.74
17	42.48
226	41.51
132	39.60
108	38.82
42	38.70
115	37.30
149	37.16
183	36.74
57	36.33
174	35.67
186	34.93
151	34.91
90	34.52
148	34.27
199	33.75
92	32.94

For the Stanford Male and Stanford Female samples, the critical value of Mahalanobis Distance which indicates a multivariate outlier is 18.08. Therefore, any case which had a Mahalanobis Distance greater than or equal to 18.08 was considered an outlier. Table 6 and Table 7 report the cases identified as outliers for the Stanford Males sample and Stanford Females sample, respectively. In total, 17 outliers were identified from the Stanford Male sample of 250 cases and 19 outliers were identified from the Stanford Female sample of 250 cases.

Table 6

Cases Identified as Outliers: Stanford Male Sample

Case Number	Mahalanobis Distance Value
49	59.34
33	37.51
1	36.12
19	33.76
40	29.58
209	29.03
15	25.93
55	25.79
129	25.18
145	24.67
234	24.32
221	24.02
61	20.74
22	19.77
210	19.30
205	18.47
115	18.27

Table 7

Cases Identified as Outliers: Stanford Female Sample

Case Number	Mahalanobis Distance Value
195	46.90
239	33.59
44	29.89
94	23.88
36	23.82
97	23.63
244	22.97
218	22.17
232	21.57
86	21.13
95	21.12
90	20.36
99	19.44
141	18.81
139	18.63
17	18.62
58	18.44
85	18.39
131	18.29

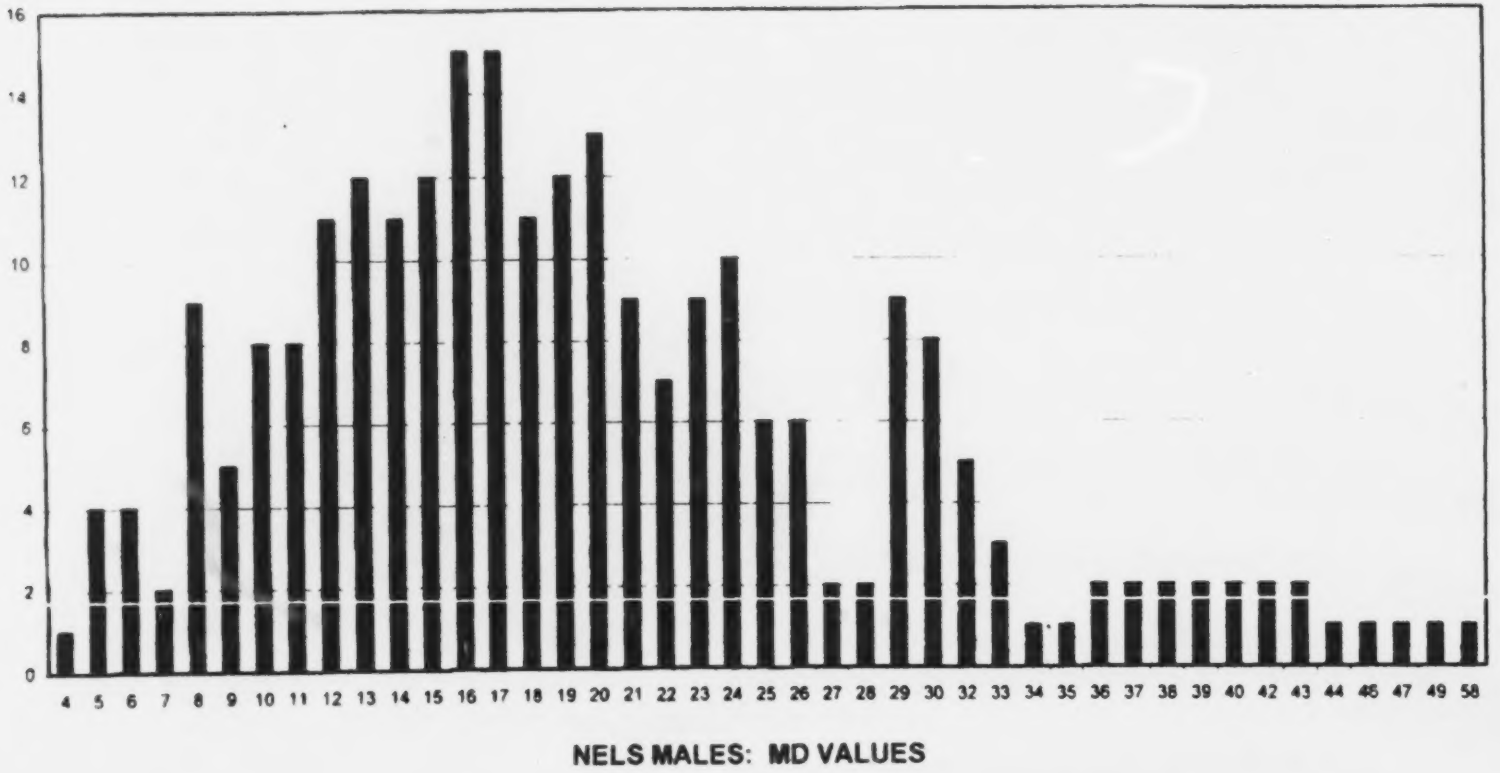
Mahalanobis' Distance Values: Histogram Displays by Sample

For the NELS-Males sample, the histogram in Figure 1 reflects Mahalanobis Distance Values ranging from a low of 3.72 to a high of 57. 64. Using the calculated Mahalanobis Distance values for the NELS samples, a histogram was constructed to display both the frequency and distribution of the Mahalanobis Distance Values.

Using the calculated Mahalanobis Distance values for the NELS samples, a histogram was constructed to display both the frequency and distribution of Mahalanobis Distance values. For the NELS-Males sample, the histogram in Figure 1 reflects Mahalanobis Distance Values ranging from a low of 3.72 to a high of 57.64. The histogram values show a marked positive skewness. Mahalanobis Distance values ranged from a low of 3.72 to a high of 55.13 for the NELS-Female sample. Looking at Figure 2 NELS-Females, we see a high positively skewed histogram.

Figures 3 and 4 show the histogram presentations for the Stanford -Males Mahalanobis Distance values and the Stanford -Females Mahalanobis Distance values, respectively. The Stanford-Males histogram shows values ranging from a low value of 0.73 to a high value of 59.34 for calculated Mahalanobis Distance values. The Stanford-Females histogram of Mahalanobis Distance values shows a range of values from a low of 0.87 to a high value of 46.90.

Figure 1 NELS Males MD Values



Note MD values have been rounded and grouped per Excel

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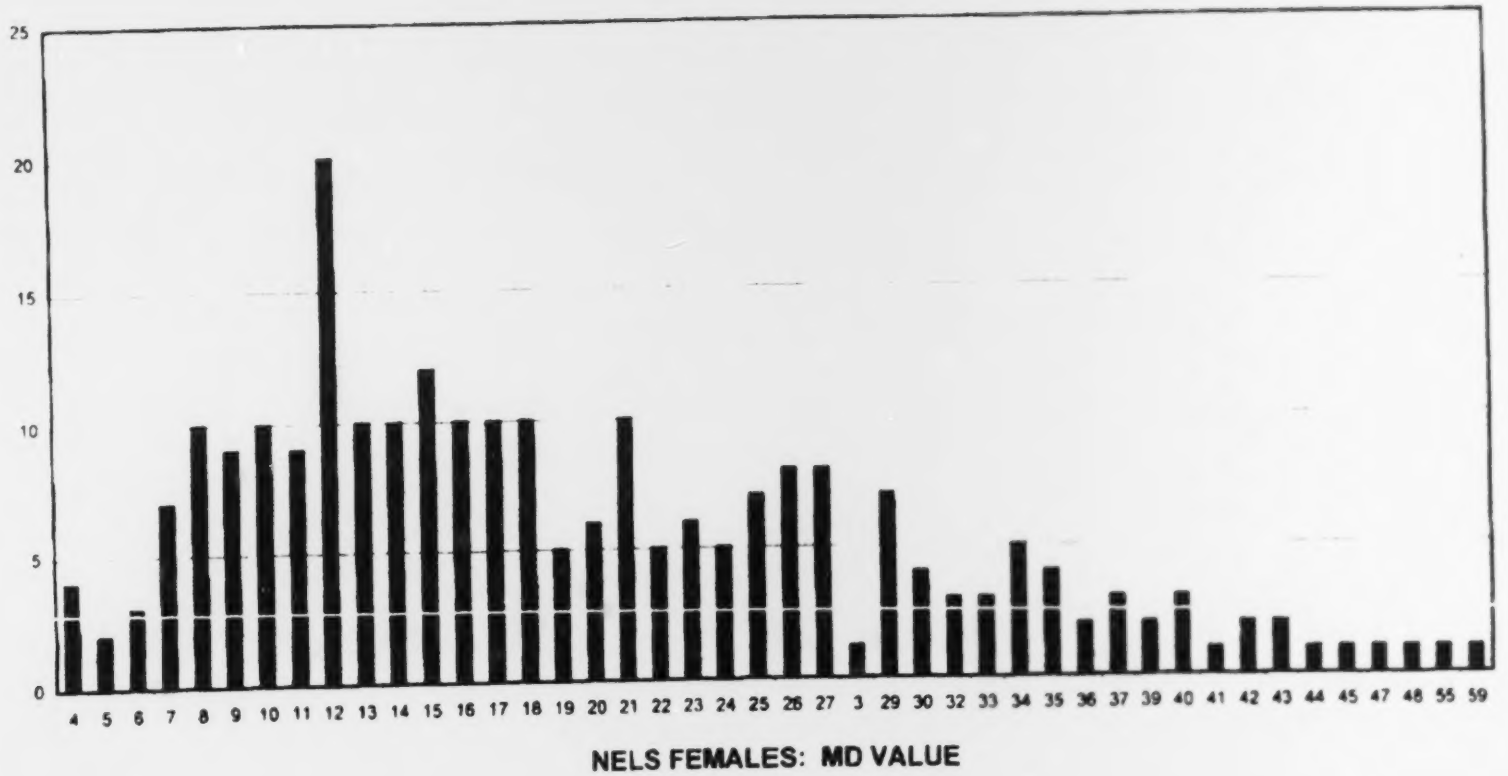
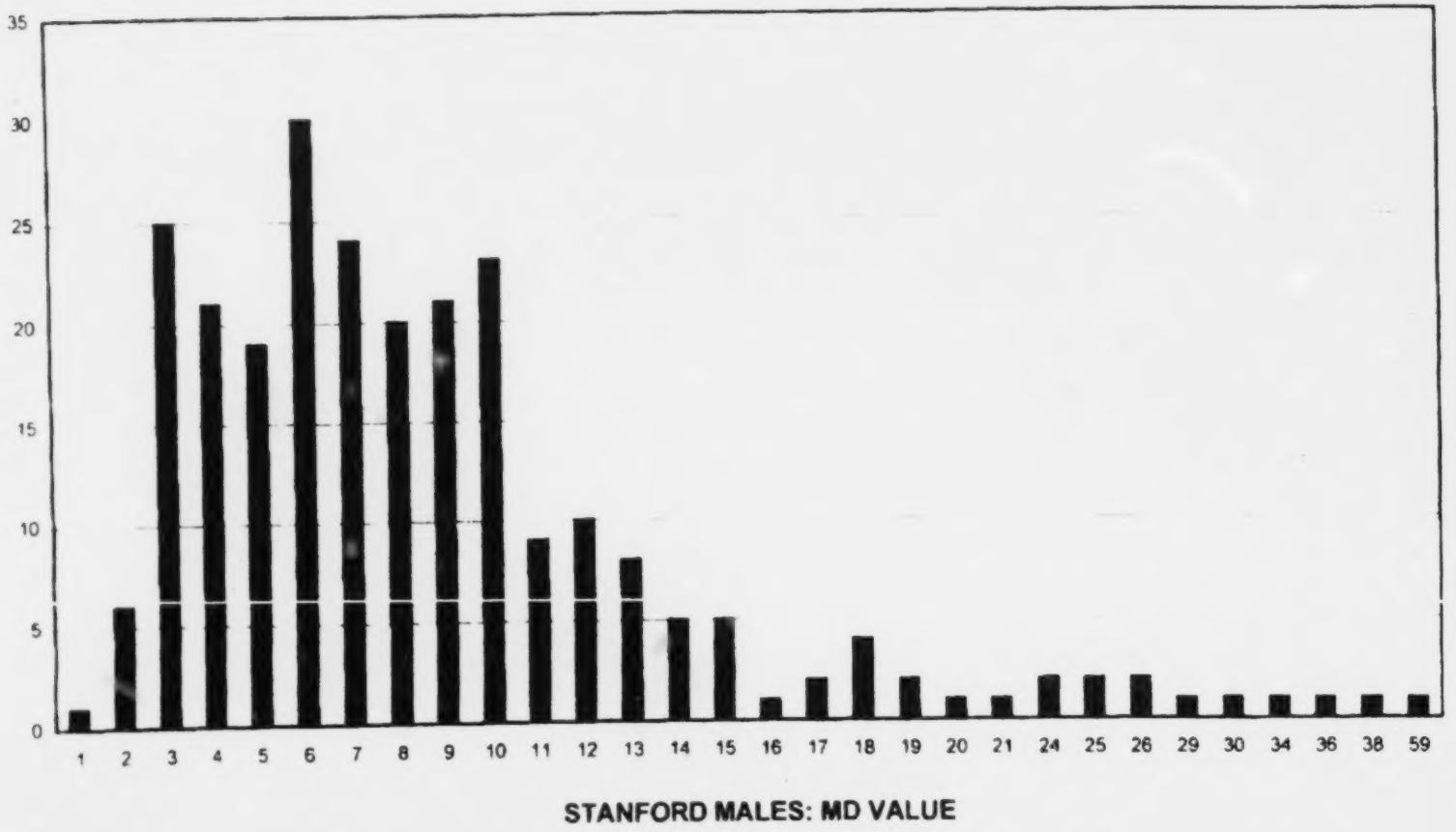
Figure 2. NELS Females: MD Values



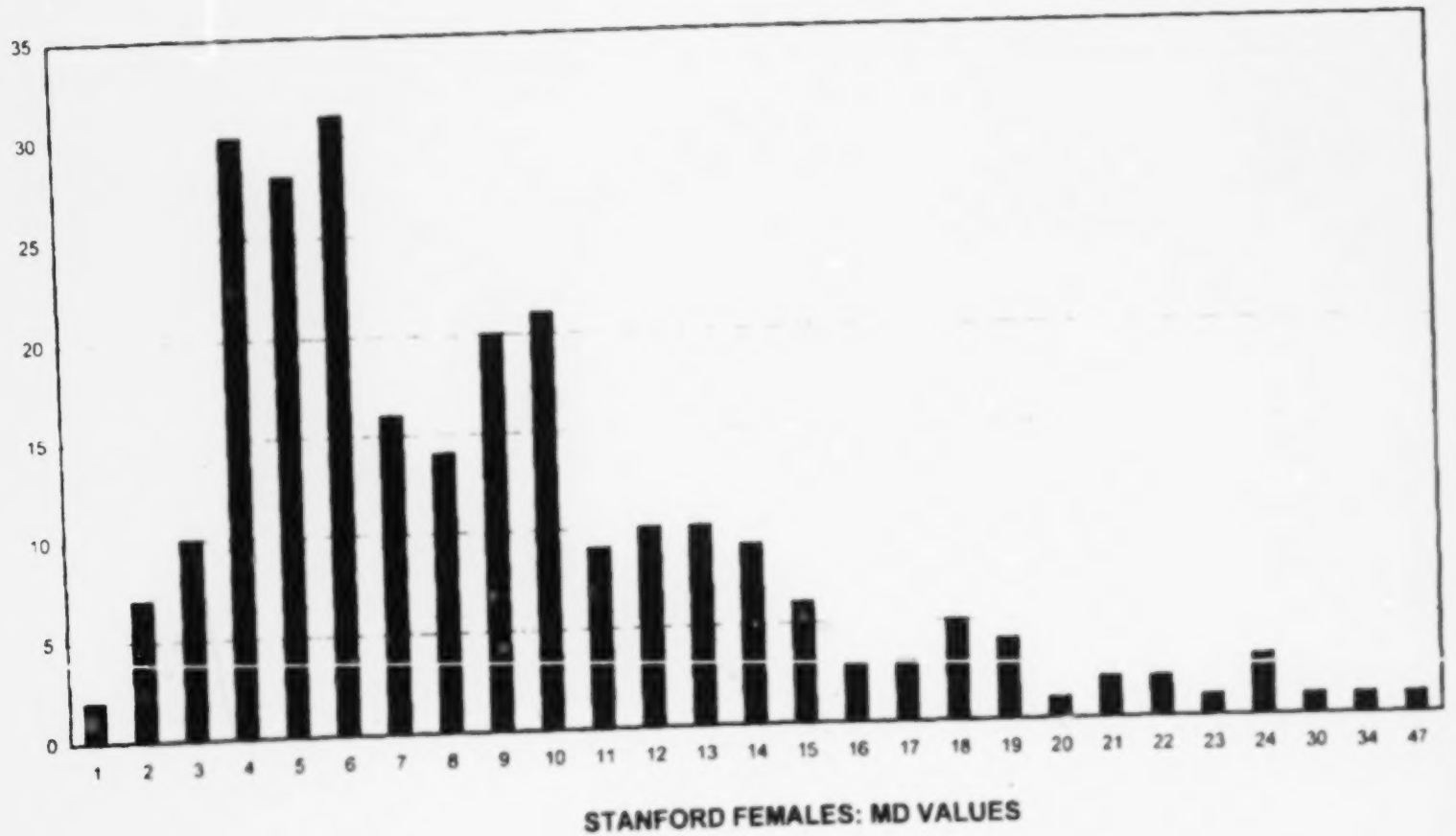
Note MD values have been rounded and grouped per Excel

Figure 3. Stanford Males: MD Value



Note MD values have been rounded and gropued per excel

Figure 4 Stanford Females: MD Values



Note MD values have been rounded and gropued per excel

Graphical Display of FACES

Using the Mahalanobis Distance Values previously calculated for each case, the cases having the largest Mahalanobis Distance Value, the smallest Mahalanobis Distance Value, and the median Mahalanobis Distance Value per sample were identified. These 3 cases (per sample) were then sequentially presented using the modified Chernoff FACES algorithms to aid in the continuity of graphical presentation.

Shown in Figure 5 are the NELS-Males and NELS-Females FACES corresponding to the largest Mahalanobis Distance Value, the smallest Mahalanobis Distance Value, and the median Mahalanobis Distance (typical case). Shown in Figure 6 are the Stanford-Males and Stanford-Females FACES corresponding to the largest Mahalanobis Distance Value, the smallest Mahalanobis Distance Value, and the median Mahalanobis Distance.

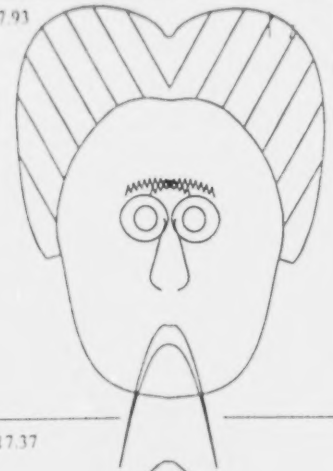
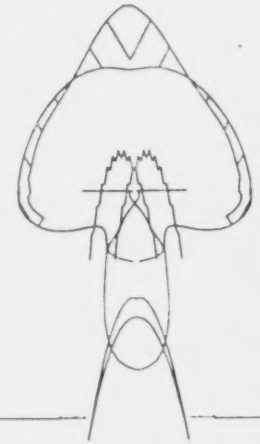
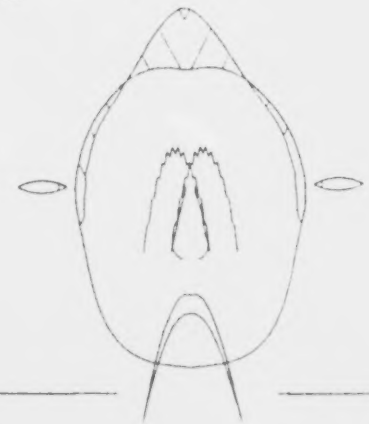
Sample	Minimum Mahalanobis Distance Value (closest to centroid)	Median Mahalonobis Distance Value (typical case)	Maximum Mahalanobis Distance Value (outlier)
NELS-Male	MD = 4.39	MD = 17.93 	MD = 57.64 
NELS - Female	MD = 3.72	MD = 17.37	MD = 55.13 

Figure 5. NELS -Samples: Baseline FACES

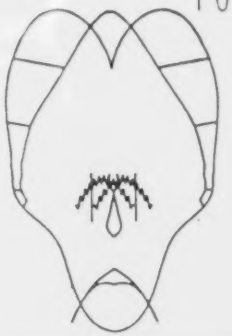
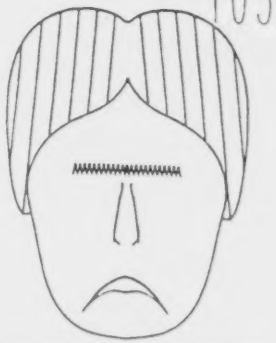
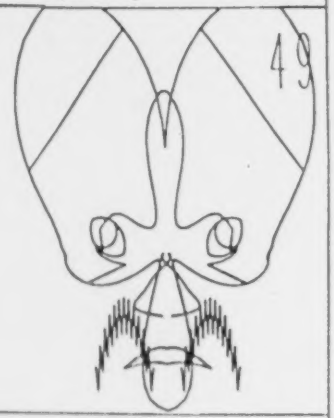
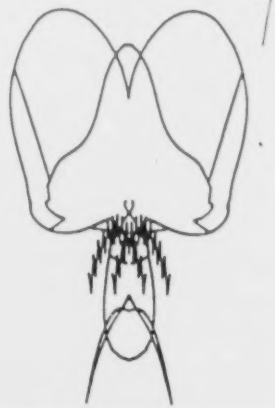
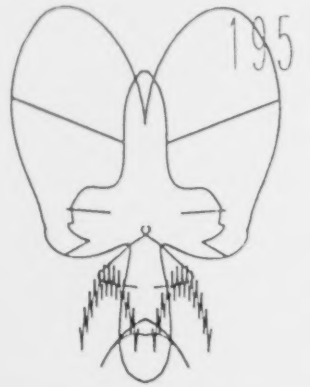
Sample	Minimum Mahalanobis Distance Value (closest to centriod)	Median Mahalonobis Distance Value (typical case)	Maximum Mahalanobis Distance Value (outlier)
Stanford Male	MD = 102 	MD = 105 	MD = 49 
Stanford Female	MD = 87	MD = 7 	MD = 195 

Figure 6. Stanford Sample: Baseline FACES

Baseline FACES: NELS Datasets

Shown in Table 8 are the graphing specifications for the first graphing of the NELS datasets. There were 250 male cases and 250 female cases for a total of 500 observations. Variable-to-feature assignments were assigned using the calculated statistical correlates for each of the NELS variables included in the dataset. These calculated correlates were used by the researcher to enable optimal variable feature allocation. The NELS-Baseline FACES variable-to-feature assignments are summarized in Table 8.

Looking at NELS-Female FACES (Figure 5), we see the FACES corresponding to the highest Mahalanobis Distance Value, (MD = 55.13) to the lowest Mahalanobis Distance Value (MD = 3.72) and the median Mahalanobis Distance Value, (MD = 17.37). The minimum FACES (case #83) shows a uniform FACE value shape, with lots of hair (UPPHA, upper hair line), with little hair shading slant (HSSL, hair shading slant), and little hair darkness (DAHA, darkness of hair). This indicates this student was very afraid to ask questions in social studies (UPHA value), not afraid at all to ask questions in science class (DAHA) and looked forward to science class (FALL face line). The FACE is uniform, while the facial features are mostly uniform in placement and size. The density of the eyebrow, DEEB, is heavy; indicating the student is often afraid to ask questions in English class.

The size of the mouth is markedly large (SIMO, size of mouth) which indicates this student comes to class quite often without their textbooks. The mouth is downturned, (COMO, curvature of mouth) which indicates this student comes to class quite often without homework.

Looking at the FACE with the median Mahalanobis distance value (MD = 17.37), we see several marked changes. First, the FACE shape is appreciably narrower and less "human" due to the low FALL (FACE line) value. This indicates the student does not look forward to science class at all. Secondly, the hair presents itself as a raised, hat-like structure due to the influence of

Table 8

NELS Baseline FACES Specifications

Variable Assignments			
Parameter	Facial Feature	Variable	Variable Label
1 (EYSE)	Eye Size	BYS60A	Respondent's ability group for mathematics
2 (PUSI)	Pupil Size	BYS60B	Respondent's ability group for science
3 (POPU)	Position of Pupil	BYS63	Friends encrg discrg Respondent from taking algebra
4 (EYSL)	Eye Slant	BYS69A	Usually look forward to mathematics class
5 (HPEY)	Horizontal position of eye	BYS69B	Afraid to ask questions in mathematics class
6 (VPEY)	Vertical position of eye	BYS69C	Mathematics will be useful in my future
7 (CUEB)	Curvature of eyebrow	BYS70A	Usually look forward to English class
8 (DEEB)	Density of eyebrow	BYS70B	Often afraid to ask questions in English class
9 (HPEB)	Horizontal position of eyebrow	BYS70C	English will be useful in my future
10 (VPEB)	Vertical position of eyebrow	BYS71A	Look forward to social studies class
11 (UPPHA)	Upper hair line	BYS71B	Afraid to ask questions in social studies
12 (LOHA)	Lower hair line	BYS71C	Social studies will be useful in my future
13 (FALI)	Face line	BYS72A	Usually look forward to science class
14 (DAHA)	Darkness of hair	BYS72B	Afraid to ask questions in science class
15 (HSSL)	Hair shading slant	BYS72C	Science will be useful in my future
16 (NOSE)	Nose line	BYS78A	How often come to class w/o pencil paper?
17 (SIMO)	Size of mouth	BYS78B	How often come to class w/o books?
18 (CUMO)	Curvature of mouth	BYS78C	How often come to class w/o homework?
	Face Color = Black		
	Hair color = Black		

UPPHA (upper hair line) having a small value, which indicates this student is not afraid to ask questions in social science class. The mouth has good size (SIMO, size of mouth) and appropriate curvature (CUMO), curvature of mouth); which tells us this student comes to class with both their textbook and their homework completed. The eyes are quite large, (EYSE, eye size), which relates that this student is in a high ability group for mathematics. Lastly, the eyebrows are horizontally shifted (HPEB, horizontal position of eyebrow), showing that this student does not believe English will be useful in their future.

Looking at the NELS-Female with the highest Mahalanobis distance value, (MD = 55.13), we see the visual results of extreme scores. They eyes have left the FACE, due to the HPEY value (horizontal position of eye); indicating this student is abjectly terrified of asking questions in math class. Again, the mouth is downturned (CUMO), indicating this student also comes to class without their homework. The eyebrows are large, long, and downturned (CUEB: curvature of eyebrow, VPEB: vertical position of eyebrow).

Baseline FACES: Stanford Datasets

Shown in Table 9 are the graphing algorithm specifications for the Stanford datasets. There were 250 male cases and 250 female cases for a total of 500 observations. Variable-to-feature assignments were allocated as described in Table 9. Using the calculated Mahalanobis Distance Values for each of the cases, the cases with the highest Mahalanobis Distance Value, the lowest Mahalanobis Distance Value and the median Mahalanobis Distance value were identified.

Table 9.

Stanford Baseline FACES Specifications

Variable Assignments		
Parameter	Facial Feature	Variable
1 (EYSI)	Eye Size	totread
2 (PUSI)	Pupil Size	totread
3 (POPU)	Position of Pupil	probsolv
4 (EYSL)	Eye Slant	probsolv
5 (HPEY)	Horizontal position of eye	mathproc
6 (VPEY)	Vertical position of eye	mathproc
7 (CUEB)	Curvature of eyebrow	lang
8 (DEEB)	Density of eyebrow	lang
9 (HPEB)	Horizontal position of eyebrow	prewrite
10 (VPEB)	Vertical position of eyebrow	prewrite
11 (UPPHA)	Upper hair line	compose
12 (LOHA)	Lower hair line	compose
13 (FALI)	Face line	editing
14 (DAHA)	Darkness of hair	editing
15 (HSSL)	Hair shading slant	useinfoA
16 (NOSE)	Nose line	useinfoA
17 (SIMO)	Size of mouth	basicbat
18 (CUMO)	Curvature of mouth	basicbat
	Face Color = Black	
	Hair color = Black	

Stanford Baseline Faces: A Visual Primer

Looking at Stanford Baseline Faces, Figure 6, we see displayed from left to right the FACES corresponding to minimum Mahalanobis distance (closest to centroid), median Mahalanobis distance (typical case), and maximum Mahalanobis distance (extreme case).

Stanford female case #87 (MD value = 0.87) exhibits the most human-like proportions and structural uniformity of the displayed FACES. Eye size (EYSI) and pupil size (PUSI) reflect high scores on the Total Reading subtest. Dense, well-placed eyebrows indicate high scores on both the Language subtest and the Pre-Writing subtest; resulting in increased eyebrow density (DUEB). The uniform eyebrow placement is controlled by HPEB, horizontal position of eyebrow, and VPEB, vertical position of eyebrow. This uniform placement indicates moderately high scores on the Mathematics Procedures subtest. The upper hair line (UPPHA) and lower hair line (LOHA) reflect moderate scores on the Basic Battery. In case #87, the mouth is full and shows a nicely-placed curvature construction due to moderately high scores on the Basic Battery subtest as reflected by size of mouth (SIMO), and curvature of mouth (CUMO).

As the Mahalanobis distance value increases to the median value, as shown by Case #7 (MD value = 7.6), we see a radical departure from the uniform features of Case #87. In Case #7, FACE shape (FALI) has changed dramatically due to a lowered score on the Editing subtest. The grossly inflated upper hair line (UPPHA) indicates the median Female case scored well on the composition subtest. The marked downturned curvature of the mouth (CUMO) indicates the median Female did not score well on the Basic Battery subtest. The horizontal postion of the eye (HPEY) and vertical positon (VPEY) are below where we would expect to see eyes on a FACE. This altered placement indicates the median Female did not score well on the mathematics procedures subtest.

In Case #195, (MD value = 46.90), the effects noted in the median Case #7 are even more distinct. The FACE shape distortion is amplified in Case #195; as is the downturned mouth curvature (CUMO) and incresased eyebrow density (DEEB). The upper hair line (FALI) is even more distorted; resembling a bug-shaped or "alien" FACE. The mouth (SIMO) and eyebrows (HPEB) and (VPEB), show unnatural placement.

To summarize, as the Mahalonobis distance values increase (depart) from the minimum value (closest to centroid), the Stanford Female facial feature also depart from normality. Looking at the Stanford Males, we notice an interesting difference in the Cases displayed as compared to the Stanford Female FACES. In the Stanford Female FACES, the lowest Mahalanobis distance (CASE #87) exhibited the most uniform FACE. However, in this Stanford Male sample, higher mean scores in Editing resulted in a more uniform FACE shape overall for Case #105, the median male case (typical case). Additionally, the downturned mouth (CUMO) for both Case #102 and Case #105 indicates neither male case scored well on the Basic Battery subtest. The minuscule eye size (EYSI) for Case #102 and Case #105 reflects low scores on the Total Reading subtest for the corresponding male case.

To contrast, in Case #49, (MD value = 59.34), we see large, well-placed eyes. This indicates this extreme male case had a very high Total Reading Score. Therefore, the closest-to-centroid Female Case #87 and farthest-from-centroid Male Case #49 had similar scores on the Total Reading. Looking at Male Case #49, we see a larger mouth (SIMO) and less mouth (downward) curvature (CUMO) than Cases 102 and 105, reflecting a higher score on the Basic Battery for this Case. All three Male Cases, #102, #105, and #49, exhibit better eye placement (HPEY, VPEY) overall as compared to Female cases #7 and #195 due to higher scores on the mathematics procedures subtest.

To summarize, the Male FACES differ from the Female cases in the following way; with the Female cases, the farther the Mahalonobis distances were from the lower value (Case #87), the more irregular the features become. To contrast, with the Male FACES, the median FACE (Case #105) became the point of departure for visual discrepancies. As the Mahalanobis Distance values moved in either direction from the median value, increasing or decreasing, the FACE features deteriorated rapidly; becoming less uniform quite quickly, thus showing the effects of the changing score values.

Educational Importance

The application of the FACES graphing algorithm to these selected student data samples creates a unique visual event: the translation of data into a meaningful visual correlate. Unlike other 2 and 3-dimensional graphical display devices, the FACES algorithm provides one summative visual image per student, rather than separate data pieces. Each students' 'FACE' is their own personal graphic representation combining up to 20 pieces of information in a highly recognizable, usable format. Each student's 'FACE' is a visual embodiment of their academic performance - a holistic summary of their individual academic strengths and weaknesses.

Aesthetically, the most integral facial component is the FACE shape. Under the Baseline FACES guidelines, the facial shape changed appreciably as the calculated Mahalanobis Distance values moved along the sample continuum of least value to greatest value. As Mahalanobis Distance Values departed in either direction from the median value, the facial shape became less "human" and more "alien-like." This greatly aided the visual identification of extreme (outlier) cases since the resulting facial shape showed how much each observed cases values departed from the median values for each sample.

Given the importance of outlier detection, the utility of Chernoff's faces as a data analysis tool cannot be underestimated. The ease of outlier detection when presented with a page of Chernoff FACES is readily apparent to the trained researcher, the school administrator and the community of parents. In contrast, a page of numbers, while containing the same original information, may be all but incomprehensible to the end consumers of educational research.

Carefully researched knowledge is useless if the information is not disseminated clearly and timely to the stakeholders in the educational system - teachers, parents, and students. Presenting information in an understandable, accessible format such as Chernoff's FACES provides an essential linkage between information and implementation.

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Appendix 1. Value of Mahalanobis Distance: NELS Male Sample

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
1	24.57		26	20.14
2	27.82		27	7.95
3	24.22		28	20.91
4	13.10		29	6.80
5	5.19		30	24.66
6	16.83		31	22.28
7	16.12		32	24.38
8	17.56		33	4.39
9	10.32		34	5.71
10	12.48		35	15.96
11	40.31		36	15.05
12	29.51		37	21.74
13	25.93		38	11.59
14	17.75		39	30.30
15	19.67		40	22.45
16	8.87		41	22.79
17	21.93		42	32.11
18	23.25		43	11.91
19	30.03		44	47.45
20	20.63		45	10.36
21	17.11		46	29.86
22	14.06		47	25.76
23	7.78		48	7.85
24	8.17		49	20.85
25	19.61		50	17.93

Appendix 1. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
51	22.71		76	4.88
52	45.75		77	9.60
53	16.67		78	19.32
54	29.07		79	18.72
55	11.57		80	24.03
56	4.88		81	14.15
57	30.41		82	18.71
58	23.70		83	12.74
59	32.78		84	31.73
60	23.23		85	5.40
61	12.58		86	35.32
62	9.48		87	15.35
63	11.02		88	15.65
64	16.82		89	10.56
65	43.02		90	13.05
66	32.47		91	20.11
67	8.16		92	7.13
68	9.91		93	22.30
69	57.64		94	32.95
70	42.51		95	29.36
71	28.34		96	14.78
72	24.06		97	16.00
73	16.45		98	42.15
74	7.76		99	39.37
75	29.70		100	36.64

Appendix 1. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
101	17.44		126	9.66
102	38.49		127	11.69
103	20.60		128	19.01
104	15.60		129	17.85
105	10.50		130	26.31
106	7.56		131	12.69
107	23.84		132	20.38
108	6.39		133	15.80
109	17.13		134	37.22
110	29.14		135	16.51
111	33.71		136	29.24
112	20.99		137	14.27
113	28.64		138	10.87
114	7.63		139	16.56
115	13.68		140	23.05
116	13.14		141	11.88
117	16.79		142	21.38
118	15.86		143	14.02
119	23.37		144	20.10
120	12.13		145	20.43
121	14.09		146	20.07
122	15.38		147	18.74
123	30.00		148	17.79
124	10.97		149	15.16
125	17.76		150	27.36

Appendix 1. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
151	20.04		176	24.94
152	17.98		177	25.49
153	42.13		178	15.80
154	21.08		179	29.09
155	26.19		180	13.22
156	8.82		181	29.87
157	18.97		182	12.02
158	16.29		183	32.33
159	30.44		184	16.91
160	19.62		185	36.28
161	14.63		186	20.21
162	17.40		187	21.85
163	13.20		188	11.06
164	18.83		189	29.27
165	35.84		190	17.78
166	9.96		191	23.79
167	13.45		192	10.93
168	10.70		193	14.54
169	33.38		194	7.63
170	17.52		195	19.68
171	49.41		196	16.53
172	20.77		197	6.21
173	22.03		198	29.50
174	23.37		199	15.34
175	39.34		200	14.45

Appendix 1. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
201	15.77		226	23.88
202	37.65		227	22.96
203	24.76		228	15.10
204	22.68		229	19.32
205	21.49		230	14.47
206	26.24		231	13.83
207	44.28		232	14.11
208	5.75		233	14.66
209	18.25		234	17.37
210	9.66		235	15.77
211	12.33		236	23.79
212	10.04		237	25.73
213	17.60		238	18.73
214	8.61		239	15.40
215	14.92		240	9.48
216	26.84		241	31.84
217	12.68		242	16.43
218	19.33		243	19.22
219	16.27		244	39.71
220	13.02		245	19.89
221	16.66		246	12.49
222	24.25		247	16.53
223	25.41		248	12.41
224	16.88		249	13.31
225	14.20		250	19.49

Appendix 2. Value of Mahalanobis Distance: NELS Female Sample

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
1	30.18		26	23.79
2	12.34		27	17.12
3	30.93		28	25.41
4	55.13		29	6.62
5	13.03		30	14.10
6	21.90		31	18.07
7	11.77		32	30.94
8	11.79		33	29.65
9	17.37		34	10.78
10	31.34		35	9.69
11	12.25		36	19.07
12	17.93		37	16.15
13	14.68		38	22.43
14	7.02		39	15.46
15	10.21		40	29.19
16	29.04		41	23.61
17	42.48		42	38.70
18	10.58		43	17.72
19	12.99		44	25.13
20	7.46		45	8.75
21	26.94		46	9.27
22	46.67		47	18.03
23	20.05		48	25.90
24	27.01		49	22.62
25	7.58		50	13.77

Appendix 2. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
51	12.27		76	4.27
52	19.78		77	8.17
53	17.53		78	12.30
54	13.17		79	19.26
55	26.38		80	15.44
56	20.36		81	6.63
57	36.33		82	13.01
58	17.01		83	3.72
59	21.20		84	15.34
60	10.46		85	26.73
61	47.78		86	26.50
62	44.46		87	17.18
63	22.06		88	9.48
64	21.89		89	15.87
65	8.75		90	34.52
66	16.48		91	9.71
67	12.28		92	32.94
68	10.19		93	18.31
69	29.04		94	25.24
70	16.74		95	32.15
71	15.06		96	40.04
72	5.16		97	28.38
73	44.87		98	10.80
74	9.85		99	16.22
75	8.49		100	6.05

Appendix 2. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
101	17.34		126	8.04
102	19.38		127	9.38
103	12.12		128	16.45
104	12.43		129	17.76
105	14.36		130	27.94
106	12.96		131	9.46
107	4.32		132	39.60
108	38.82		133	5.81
109	26.40		134	14.41
110	14.75		135	40.98
111	22.63		136	31.97
112	21.07		137	30.19
113	42.74		138	14.21
114	14.11		139	9.93
115	37.30		140	21.58
116	20.21		141	15.61
117	10.76		142	16.10
118	26.79		143	8.12
119	20.06		144	43.03
120	15.30		145	12.27
121	15.41		146	18.05
122	25.94		147	11.66
123	29.69		148	34.27
124	26.47		149	37.16
125	21.35		150	21.32

Appendix 2. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
151	34.91		176	16.91
152	14.16		177	8.16
153	7.61		178	11.64
154	10.78		179	21.19
155	7.42		180	18.81
156	11.18		181	15.74
157	8.08		182	58.86
158	12.48		183	36.74
159	4.66		184	13.14
160	32.53		185	23.27
161	40.42		186	34.93
162	20.28		187	30.96
163	15.18		188	13.14
164	12.42		189	10.97
165	7.16		190	13.63
166	12.47		191	20.71
167	21.28		192	24.01
168	23.19		193	18.54
169	11.61		194	22.93
170	16.65		195	16.02
171	13.24		196	25.23
172	20.77		197	14.50
173	22.69		198	15.20
174	35.67		199	33.75
175	10.49		200	28.66

Appendix 2. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
201	15.20		226	41.51
202	33.54		227	12.62
203	11.05		228	16.69
204	11.66		229	33.52
205	21.46		230	26.71
206	28.45		231	23.73
207	28.70		232	16.22
208	26.12		233	15.08
209	9.43		234	8.90
210	25.57		235	11.40
211	12.23		236	11.53
212	4.45		237	17.70
213	24.73		238	13.40
214	25.41		239	27.16
215	33.47		240	9.26
216	32.13		241	8.28
217	6.87		242	11.94
218	8.03		243	25.90
219	6.41		244	33.69
220	10.30		245	23.84
221	9.61		246	34.85
222	25.31		247	14.31
223	21.28		248	18.11
224	29.30		249	26.93
225	29.05		250	17.41

Appendix 3. Values of Mahalanobis Distance: Stanford Males Sample

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
1	36.12		26	15.32
2	9.09		27	3.06
3	13.31		28	6.03
4	7.58		29	9.13
5	9.87		30	8.21
6	14.62		31	6.33
7	3.31		32	5.69
8	9.70		33	37.51
9	2.96		34	2.69
10	4.03		35	4.19
11	10.10		36	10.90
12	12.16		37	8.85
13	9.95		38	8.38
14	4.07		39	12.44
15	25.93		40	29.58
16	9.72		41	2.73
17	8.28		42	3.47
18	3.08		43	3.61
19	33.76		44	9.23
20	1.90		45	4.68
21	5.91		46	2.51
22	19.77		47	9.13
23	4.43		48	10.65
24	9.66		49	59.34
25	4.81		50	5.82

Appendix 3. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
51	7.58		76	8.76
52	12.56		77	3.92
53	3.42		78	6.22
54	2.89		79	9.03
55	25.79		80	14.25
56	7.14		81	6.32
57	5.33		82	3.43
58	13.03		83	4.88
59	5.09		84	4.18
60	1.92		85	5.24
61	20.74		86	4.79
62	9.95		87	7.32
63	7.63		88	9.88
64	5.41		89	6.37
65	10.14		90	7.93
66	3.69		91	8.88
67	14.40		92	5.57
68	13.64		93	11.37
69	7.95		94	7.37
70	8.64		95	2.94
71	14.40		96	7.74
72	3.29		97	6.87
73	9.33		98	11.85
74	14.40		99	4.66
75	5.11		100	8.26

Appendix 3. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
101	15.16		126	5.95
102	0.73		127	6.07
103	2.23		128	4.63
104	14.55		129	25.18
105	7.43		130	5.59
106	11.03		131	11.00
107	17.93		132	9.92
108	6.42		133	5.60
109	7.75		134	2.93
110	4.13		135	11.37
111	8.05		136	11.00
112	12.23		137	11.57
113	5.96		138	6.69
114	4.05		139	9.66
115	18.27		140	6.28
116	6.56		141	5.23
117	4.85		142	4.15
118	6.16		143	9.31
119	6.05		144	7.89
120	5.30		145	24.67
121	2.26		146	6.24
122	7.00		147	3.16
123	7.31		148	7.42
124	7.88		149	3.75
125	4.30		150	5.55

Appendix 3. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
151	12.56		176	11.73
152	9.13		177	6.59
153	7.40		178	8.37
154	9.12		179	6.38
155	9.39		180	5.67
156	3.24		181	9.92
157	5.47		182	9.23
158	10.57		183	7.79
159	9.55		184	6.18
160	12.53		185	9.64
161	9.15		186	4.13
162	17.74		187	6.51
163	5.35		188	8.70
164	14.72		189	3.91
165	3.60		190	3.25
166	10.19		191	10.49
167	10.15		192	5.87
168	2.94		193	8.38
169	8.06		194	7.20
170	6.85		195	2.89
171	8.77		196	17.09
172	1.81		197	4.06
173	6.47		198	6.54
174	6.45		199	4.12
175	11.94		200	12.87

Appendix 3. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
201	8.00		226	6.29
202	11.41		227	5.63
203	2.26		228	2.98
204	6.69		229	3.59
205	18.47		230	7.18
206	5.94		231	10.48
207	4.43		232	6.84
208	8.21		233	3.24
209	29.03		234	24.32
210	19.30		235	11.93
211	12.92		236	3.60
212	2.93		237	6.78
213	12.45		238	6.95
214	16.23		239	6.98
215	11.00		240	11.53
216	16.74		241	4.81
217	7.12		242	5.02
218	10.39		243	12.76
219	9.40		244	6.03
220	6.91		245	9.57
221	24.02		246	10.20
222	10.14		247	3.45
223	9.04		248	2.91
224	3.18		249	9.55
225	7.61		250	10.61

Appendix 4. Values of Mahalanobis Distance: Stanford Females Sample

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
1	7.05		26	11.26
2	6.67		27	4.60
3	7.70		28	5.36
4	5.66		29	5.81
5	9.13		30	4.53
6	9.06		31	12.80
7	7.60		32	11.83
8	3.55		33	11.26
9	6.00		34	10.12
10	4.01		35	7.77
11	3.47		36	23.82
12	12.70		37	5.74
13	8.52		38	5.06
14	16.18		39	6.86
15	6.96		40	1.90
16	10.27		41	5.57
17	18.62		42	2.49
18	10.30		43	12.14
19	7.16		44	29.89
20	4.80		45	5.97
21	15.04		46	5.75
22	2.78		47	6.81
23	6.40		48	11.86
24	4.35		49	10.13
25	5.78		50	4.59

Appendix 4. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
51	9.84		76	9.69
52	12.33		77	10.23
53	5.75		78	8.02
54	3.92		79	3.73
55	2.66		80	10.53
56	10.47		81	5.35
57	6.79		82	12.13
58	18.44		83	17.41
59	1.68		84	11.81
60	13.40		85	18.39
61	16.84		86	21.13
62	6.16		87	0.87
63	8.83		88	8.33
64	4.62		89	15.08
65	4.48		90	20.36
66	8.21		91	4.27
67	10.46		92	9.93
68	6.76		93	4.15
69	6.02		94	23.88
70	6.06		95	21.12
71	13.87		96	10.20
72	3.61		97	23.63
73	13.47		98	4.93
74	7.26		99	19.44
75	6.76		100	5.07

Appendix 4. (continued)

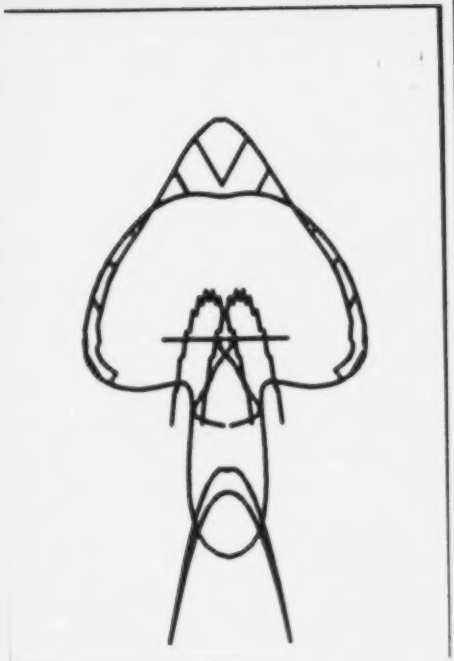
Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
101	13.73		126	6.33
102	4.29		127	14.00
103	8.74		128	11.40
104	6.00		129	10.36
105	4.41		130	9.20
106	5.00		131	18.29
107	6.69		132	9.72
108	10.73		133	3.46
109	5.53		134	2.55
110	6.43		135	5.21
111	11.27		136	14.84
112	8.78		137	5.01
113	8.75		138	2.11
114	16.73		139	18.63
115	4.72		140	4.27
116	3.28		141	18.81
117	4.83		142	5.56
118	4.06		143	12.80
119	3.86		144	15.12
120	8.93		145	3.64
121	14.48		146	2.19
122	3.92		147	6.05
123	8.74		148	8.69
124	3.68		149	6.50
125	11.86		150	10.60

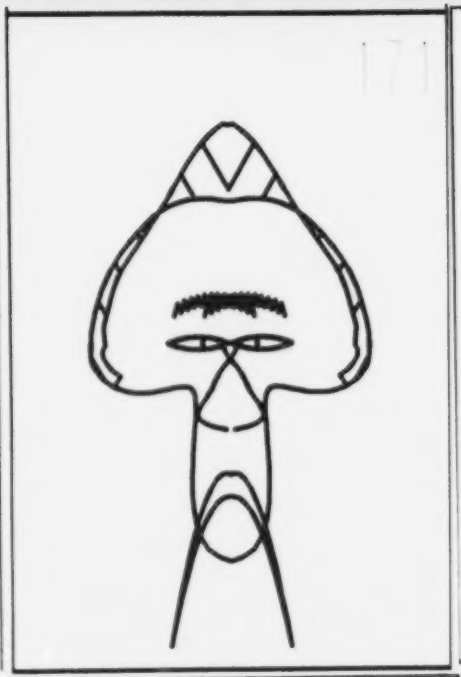
Appendix 4. (continued)

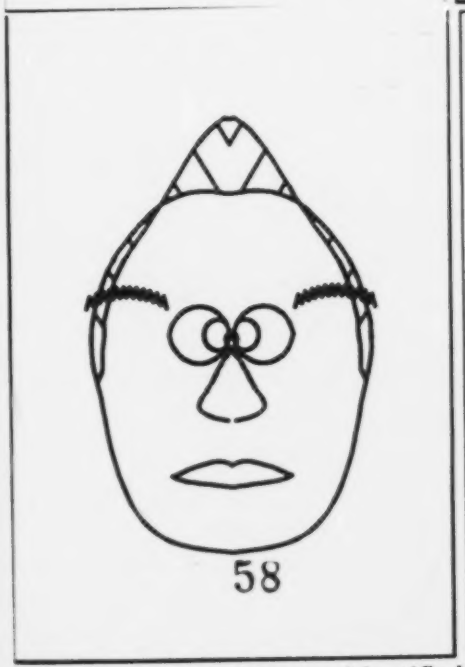
Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
151	7.97		176	9.79
152	9.20		177	1.74
153	4.88		178	6.08
154	5.92		179	4.60
155	8.33		180	7.91
156	1.67		181	13.86
157	4.20		182	5.58
158	4.18		183	5.64
159	13.94		184	5.72
160	4.23		185	3.22
161	9.15		186	2.91
162	14.74		187	5.90
163	8.19		188	8.09
164	12.66		189	17.59
165	10.25		190	3.79
166	4.56		191	9.65
167	7.29		192	16.49
168	12.83		193	5.84
169	17.64		194	14.01
170	4.34		195	46.90
171	5.06		196	15.72
172	9.96		197	4.36
173	7.03		198	4.59
174	12.95		199	9.03
175	5.16		200	6.93

Appendix 4. (continued)

Case Number	Mahalanobis Distance		Case Number	Mahalanobis Distance
201	5.61		226	10.72
202	4.84		227	3.60
203	4.70		228	9.58
204	8.98		229	7.77
205	8.57		230	3.17
206	4.17		231	10.79
207	11.63		232	21.57
208	2.62		233	4.07
209	12.57		234	8.17
210	6.88		235	5.79
211	13.52		236	7.09
212	11.39		237	15.08
213	5.27		238	14.31
214	5.31		239	33.59
215	6.25		240	5.47
216	4.80		241	5.50
217	9.91		242	8.90
218	22.17		243	9.33
219	1.43		244	22.97
220	4.23		245	3.91
221	9.63		246	8.12
222	5.69		247	12.29
223	3.67		248	12.32
224	13.19		249	8.57
225	8.62		250	3.90







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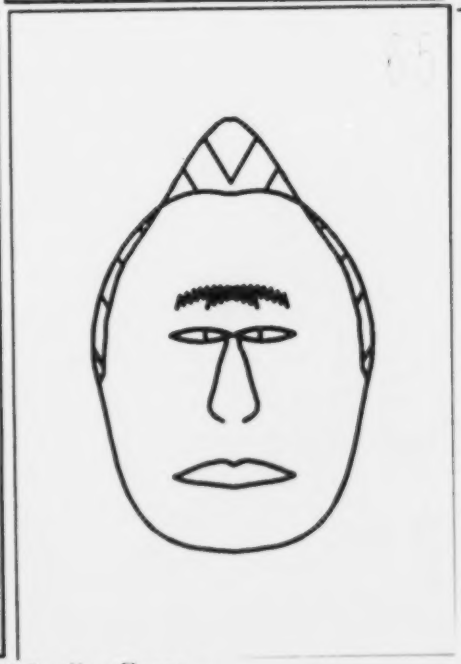

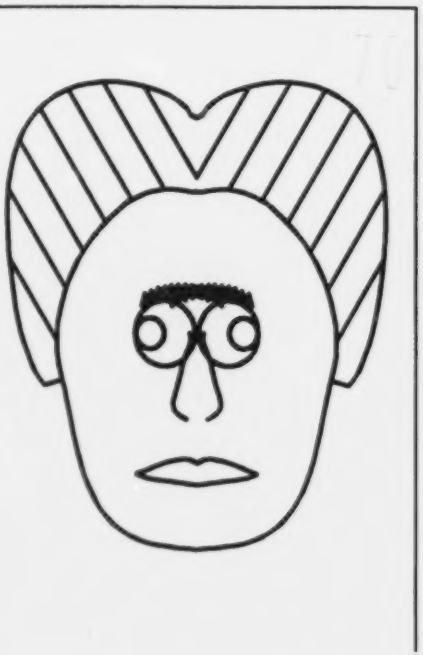

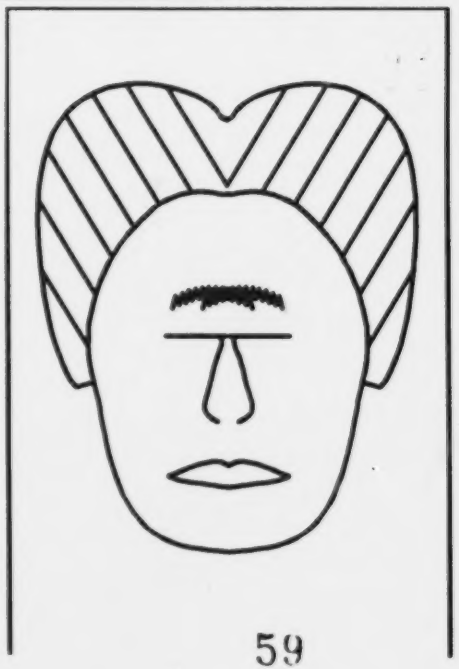

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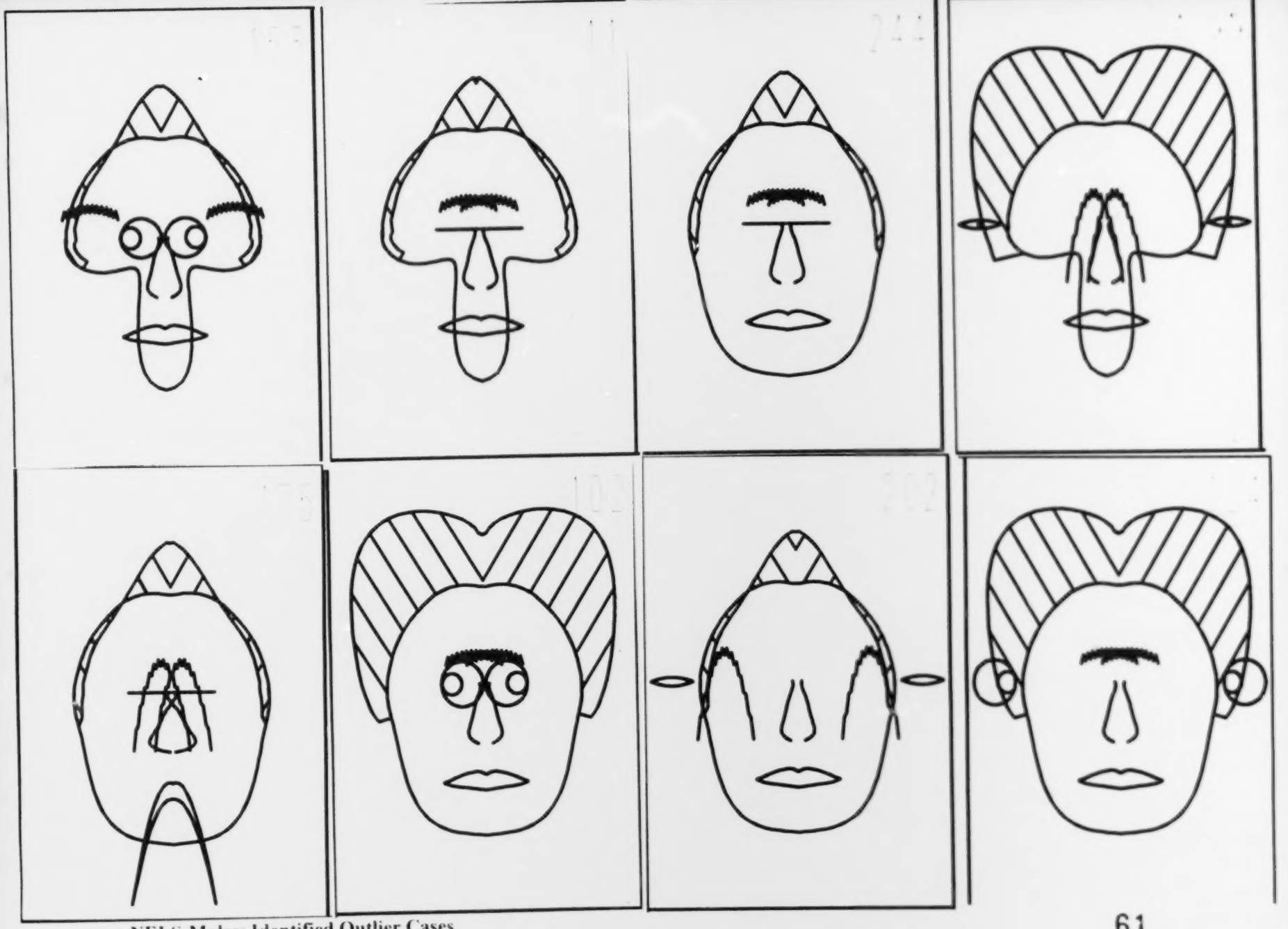
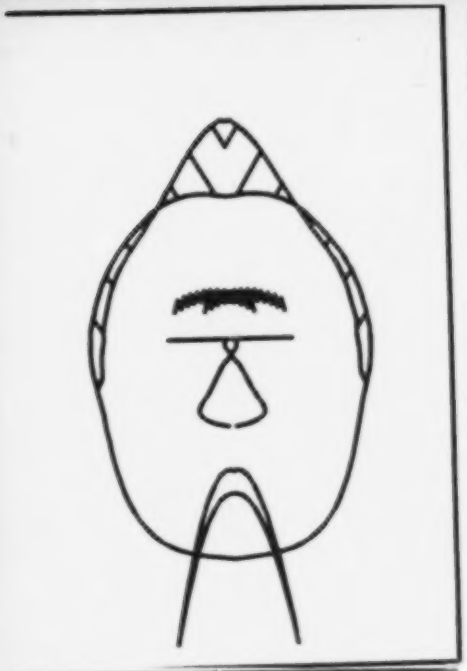
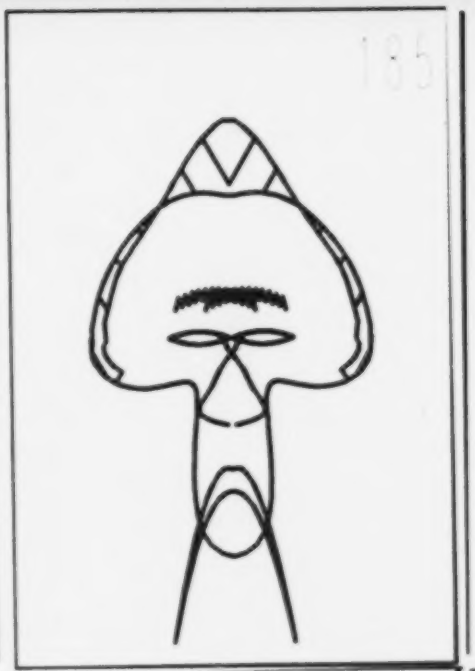
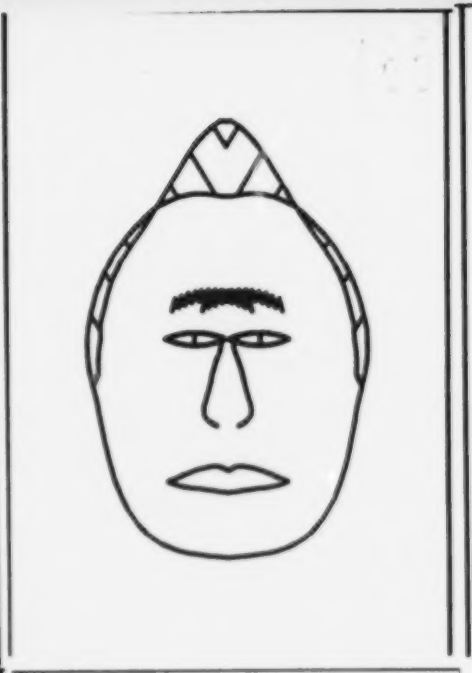
NELS-Males: Identified Outlier Cases

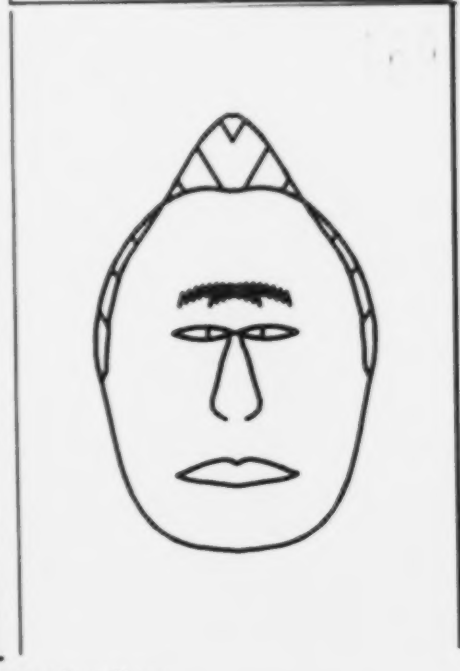


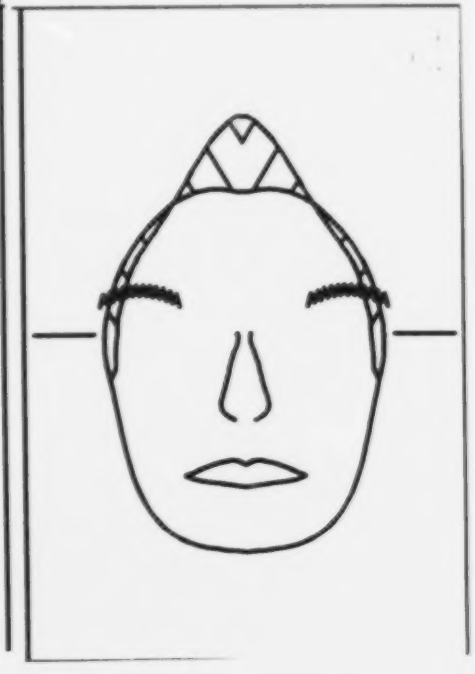
NELS-Males: Identified Outlier Cases











NELS-Males: Identified Outlier Cases

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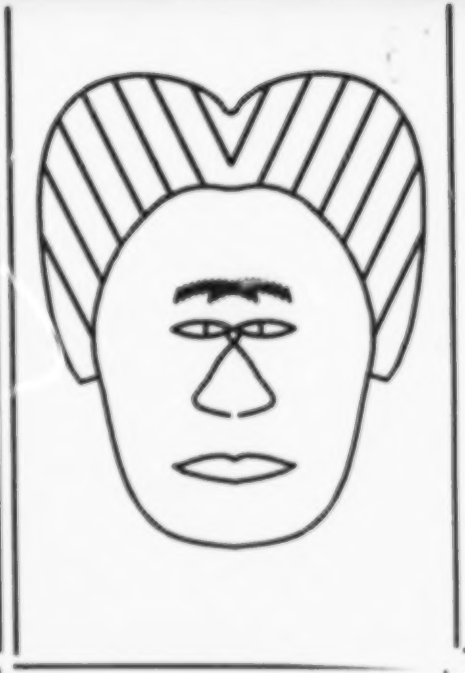

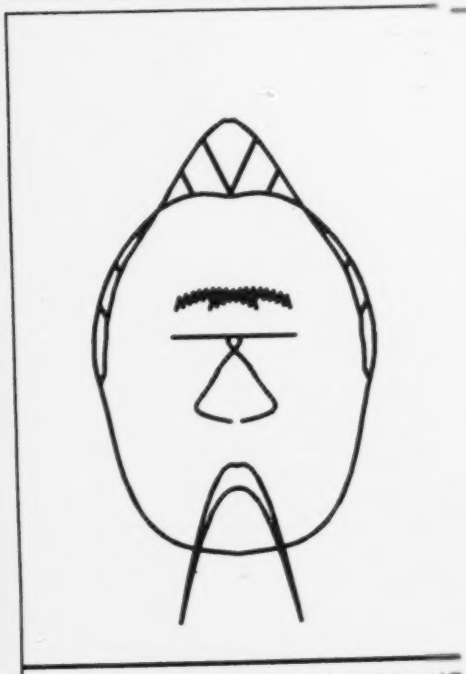

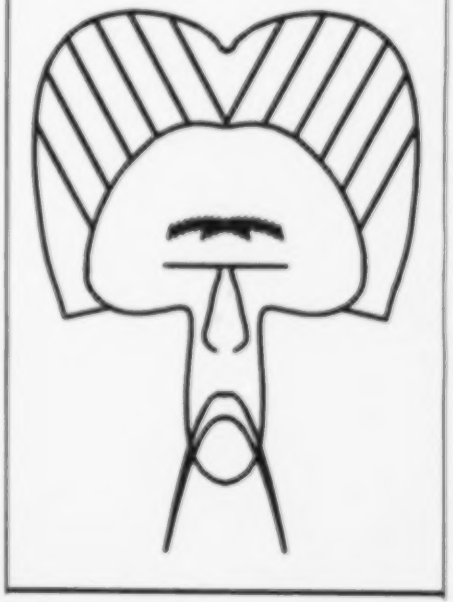

NELS-Females: Identified Outlier Cases

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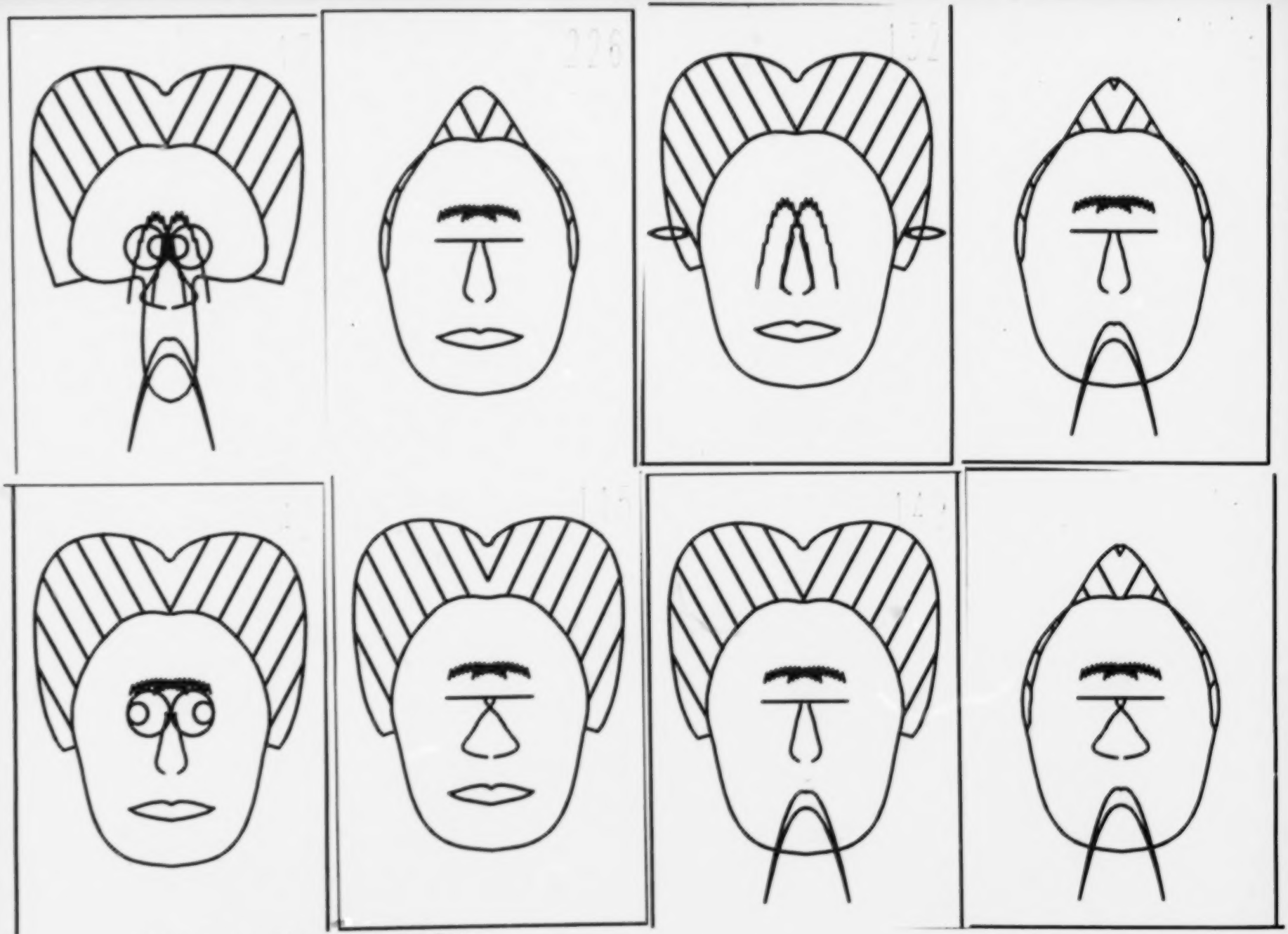

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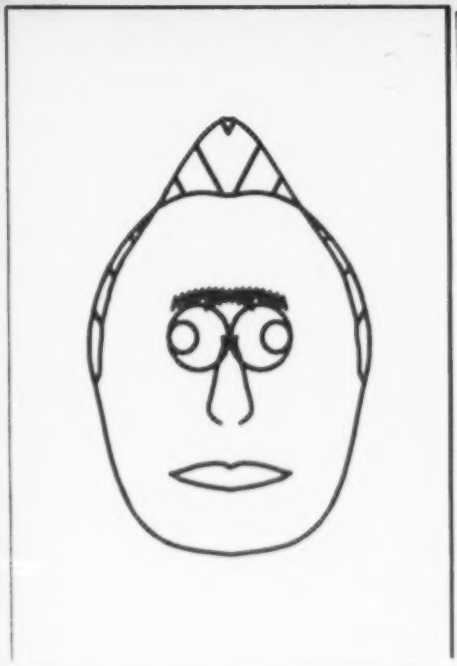
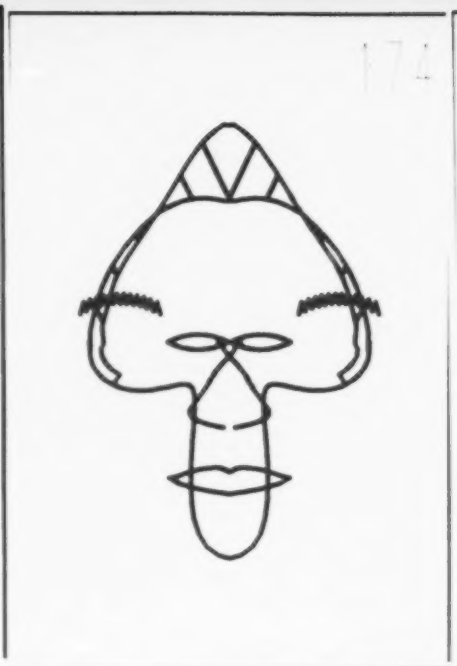
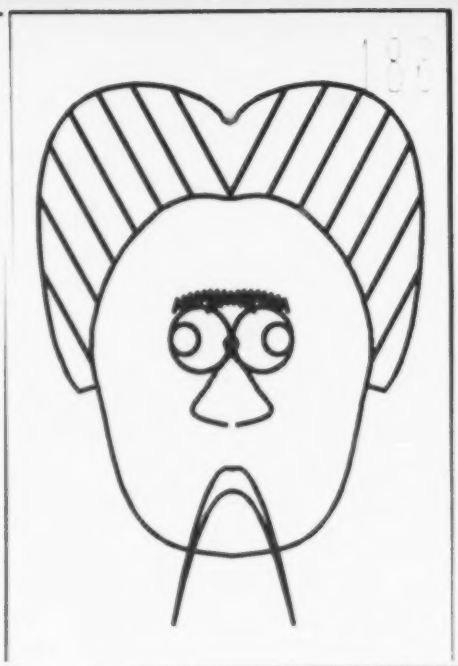
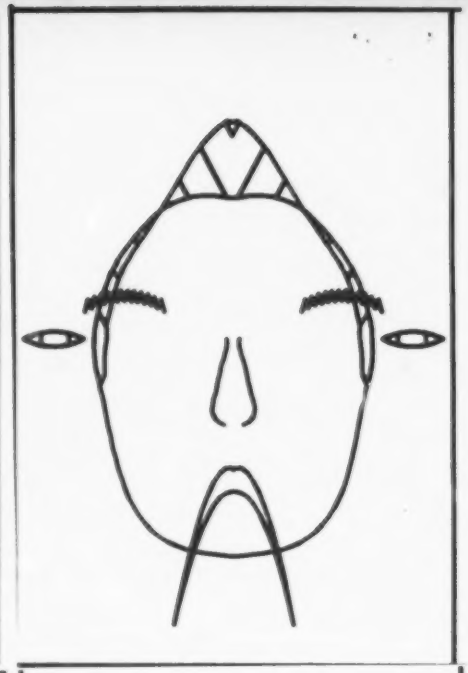
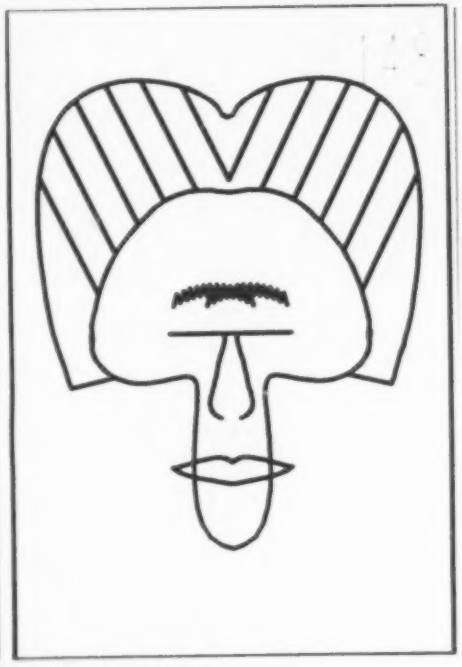
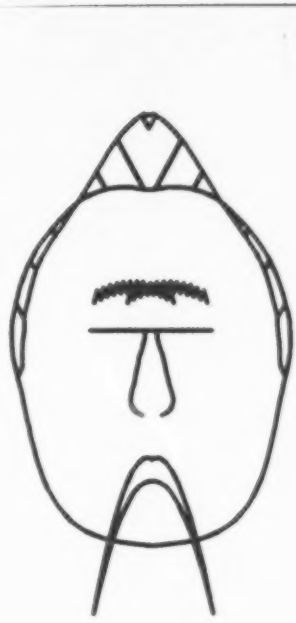


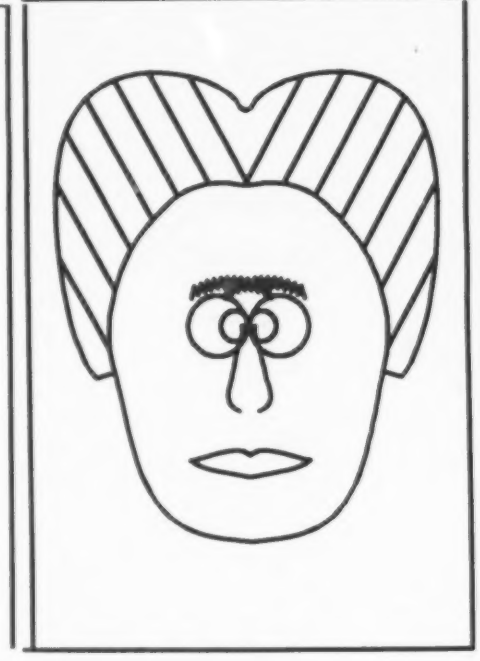






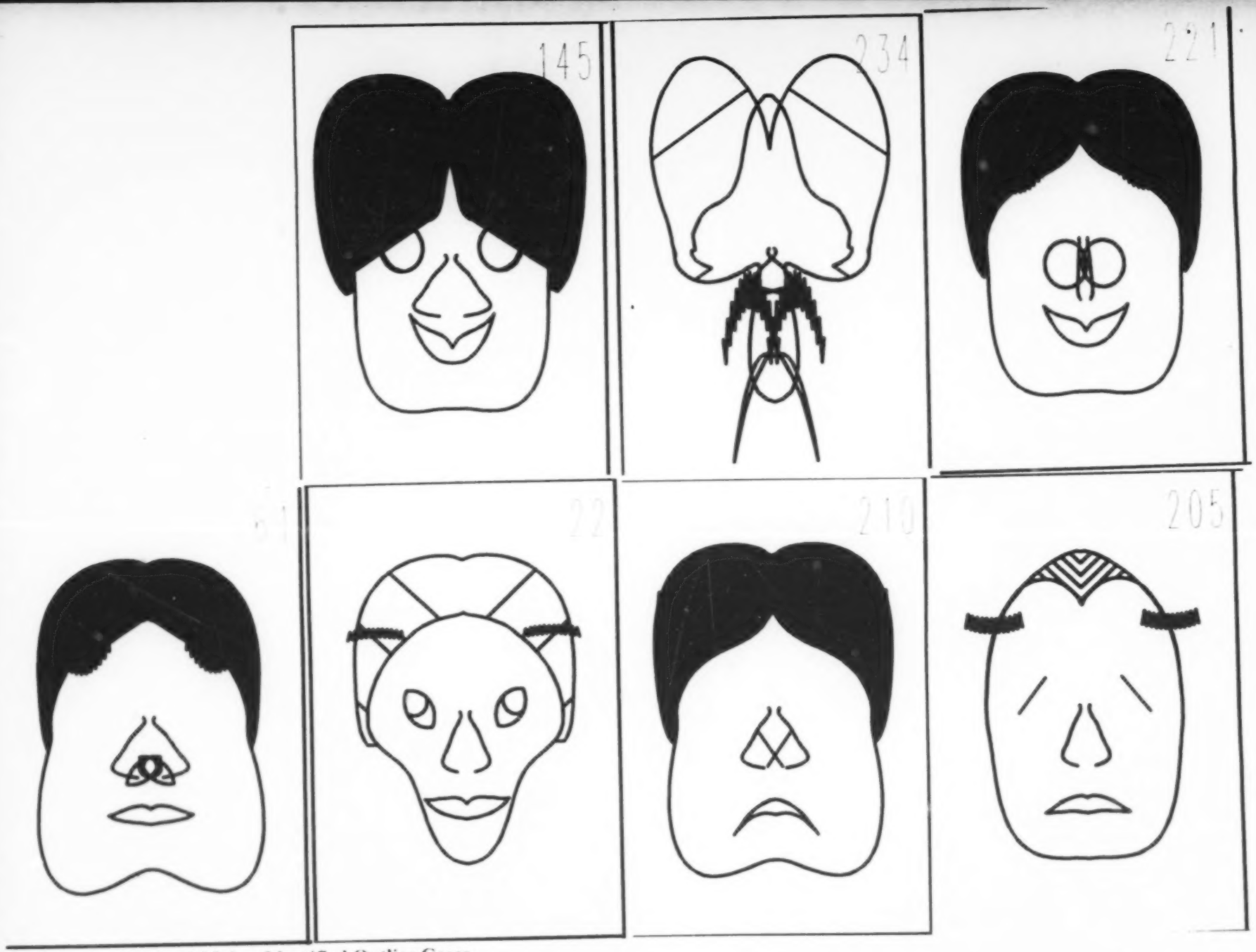




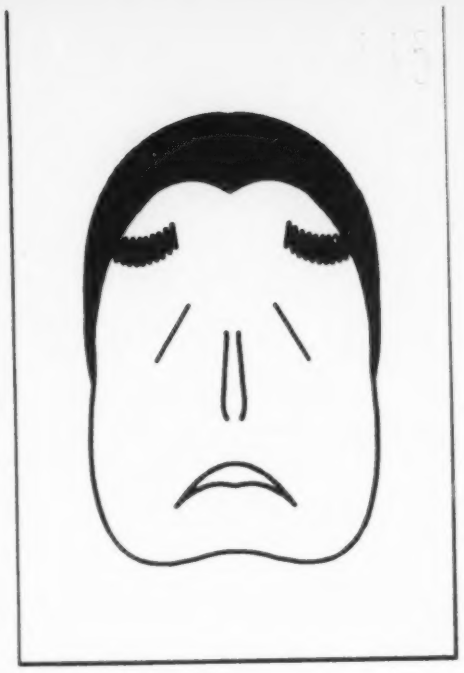


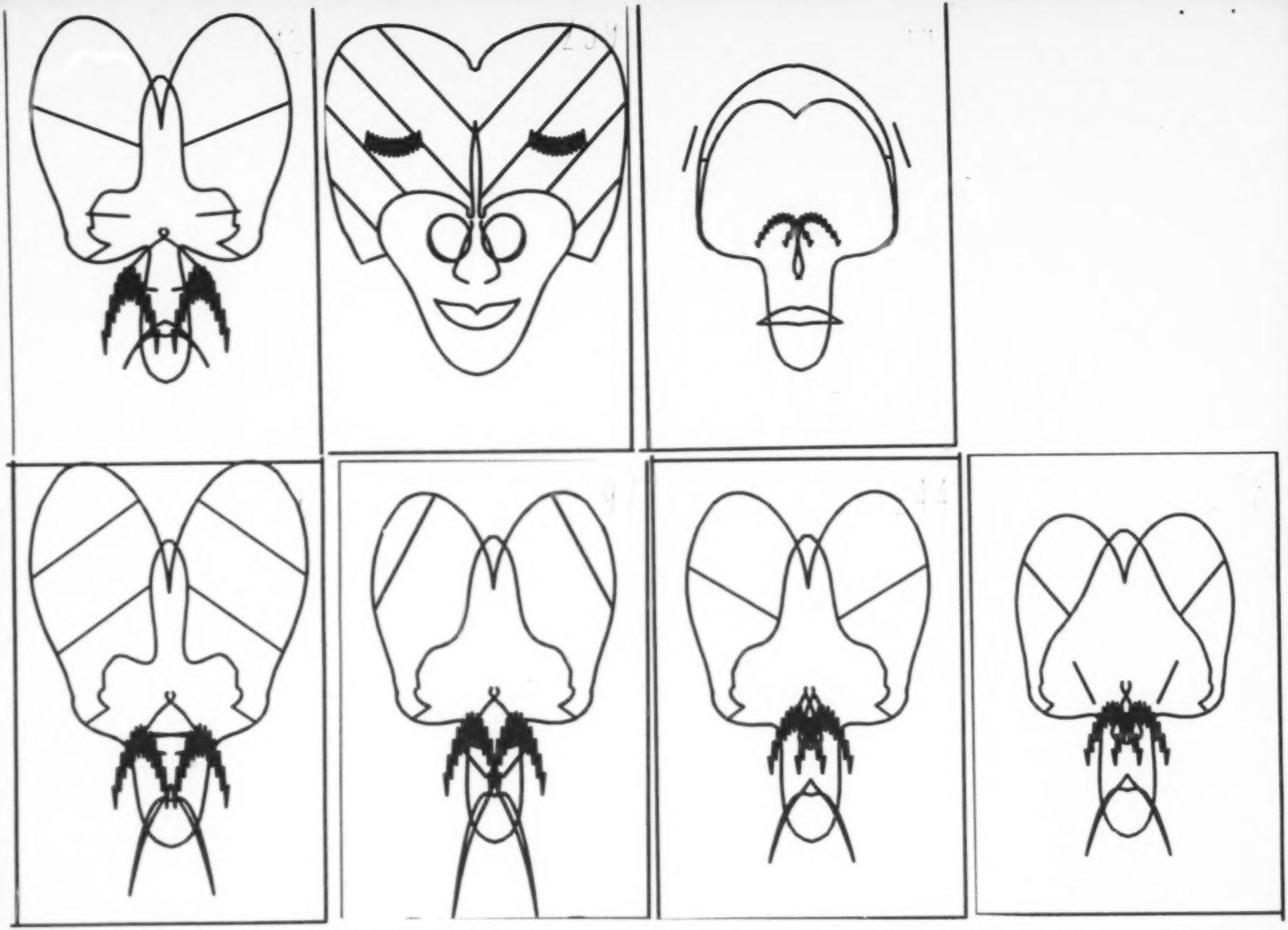
NELS-Females: Identified Outlier Cases

Stanford-Males: Identified Outlier Cases

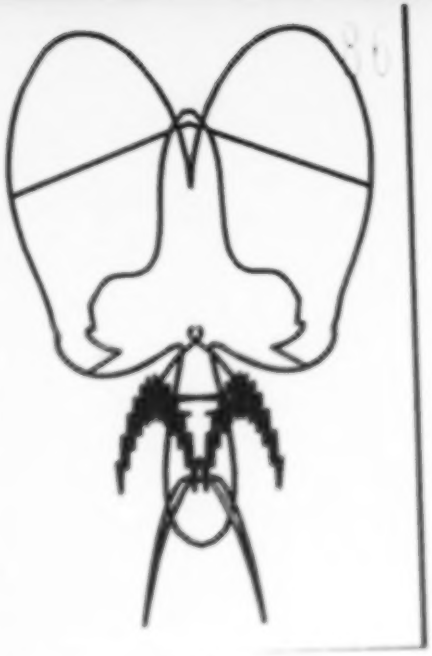


Stanford-Males: Identified Outlier Cases





Stanford-Females: Identified Outlier Cases



Stanford-Females: Identified Outlier Cases

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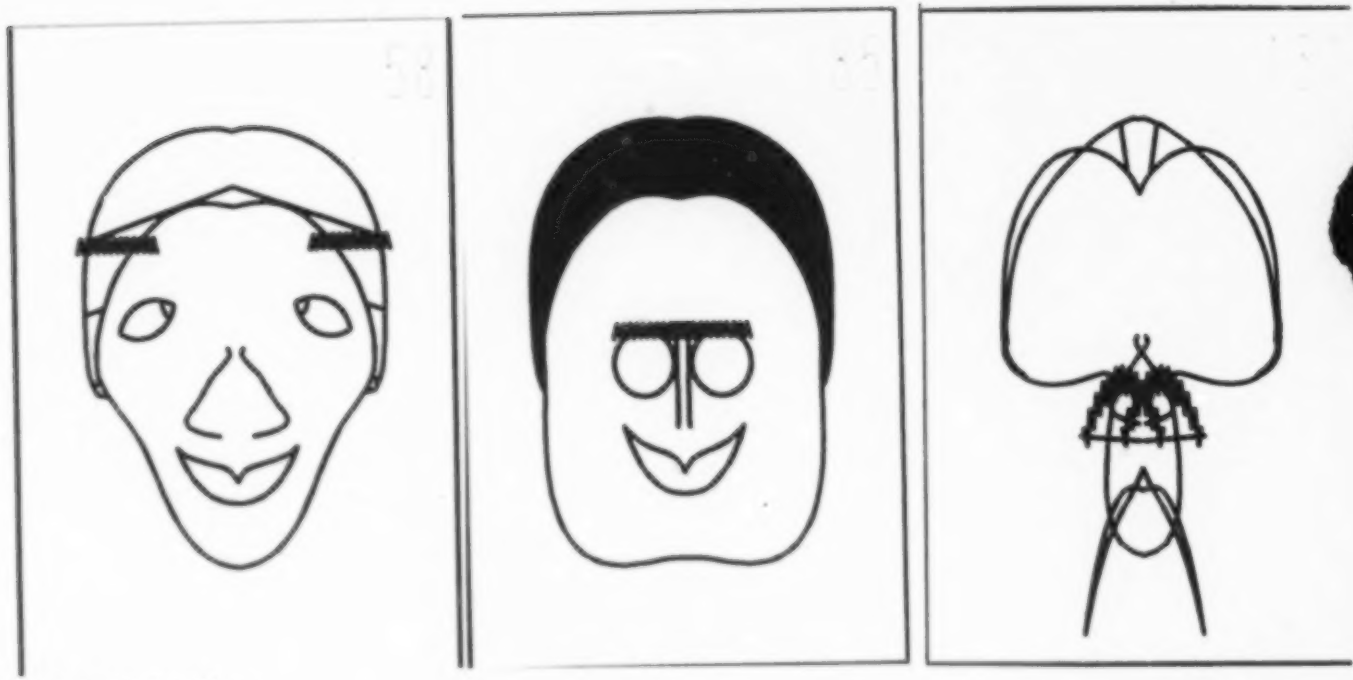

Stanford-Females: Identified Outlier Cases